Some Theoretical Analogies in the Description of Strongly Coupled Quark-Gluon Plasma

Rakesh Kumar, G.L. Sawhney

Abstract— We study some theoretical aspects of strongly coupled quark gluon plasma, as a function of Coulomb coupling parameter Γ. We study the evaluation of dynamic properties of the BCS/BEC crossover in relativistic fluid. We also discuss some fluid dynamic aspects of the crossover at critical temperature. The shear viscosity is minimum near around the unitary limit. We have clarified the thermal fluctuations in strongly coupled characteristic of plasma. The color deconfinement phase of quark gluon plasma also discussed.

Index Terms— Quark Gluon Plasma, Heavy Ion Collision, Strong Coupling, Unitary Limit.

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1 Introduction

The beautiful RHIC data that have been collected over the last few years have somehow shaken our hope that the idealized state of matter can be observed in nucleus collisions. The study of ultra relativistic heavy ion collisions offers the possibility concerning for instant the state of matter called quark gluon plasma at very high temperature and density. We propose to consider strongly coupled classical non-relativistic plasmas to learn about qualitative features of the strongly coupled quark gluon plasma. Such plasmas are well studied experimentally as well as theoretically [1]. Using magnetic field, one can use the so called Feshbach resonances and make a pair of atoms nearly degenerate with their bound state, usually called a cooper pair. This results, in so large scattering length a of the atoms, that a qualitative new type of matter, strongly coupled fermions and bose gases- is observed. This very dilute system starts to behave thermodynamically, displaying elliptically flow very similar to that in non-central heavy ion collisions gives very small value of shear viscosity. A small negative scattering length corresponds to a weak attractive interaction between the atoms. This case is known as BCS limit. With increasing strength of interaction between the atoms, the scattering length correspondingly increases and diverges at a point where a bound state is formed. This point $a = \infty$ is called unitarity limit, the limit of crossover to BEC regime. In Bose-Einstein Condensation limit (BEC-limit), the interaction is strongly attractive and fermions formed strongly bound molecules. In this limit, the atoms form a strongly coupled quantum liquid. In BCS limit, the atomic gas is characterized by the parameter $k_f a$, where k_f is the Fermi momentum. In the unitarity limit, this parameter is infinite and the system is strongly couple.

Author: Rakesh Kumar is a Research Scholar, Department of Physics, NIMS University, Jaipur, Rajasthan, India.
Email: rakesh_du144@yahoo.in

Co-Author: Dr. G.L. Sawhney is Associate Professor, Department of Physics, ARSD College, Delhi University, India.
Email: gulsawhney@yahoo.com

Most plasmas in nature and in the laboratory are non-relativistic, classical, weakly coupled (ideal) plasmas. As a matter of fact, strongly coupled plasmas are hard to produce, since they require low temperatures and high densities, at which a strong recombination sets in. In nature, only "white dwarfs" can have the ion components in these conditions. On the other hand, there is the possibility to have dense plasmas in the relativistic and ultra-relativistic nuclear collision. By definition, plasma is a quasi-neutral gas of charged and neutral particles which exhibits collective behavior. At sufficiently high temperature, near the critical value T_c, neutral particles will be negligibly small so that one can see collective plasma. A non-relativistic plasma is strongly coupled if the interaction energy (Coulomb energy) between the particles is larger than the thermal energy of the plasma particles i.e. if the Coulomb energy parameter $\Gamma = (q^2/dT) > 1$, where q is the charge of the particles, d is the interparticle distance and T is the plasma temperature. For typical temperatures, in heavy ion collisions, Γ is larger than 1. The Coulomb coupling parameter of the QGP is analogy as $\Gamma = C\, g^2/dT$ [2], where C is the Casimir invariant. For quarks C = 4/3 and for gluons C = 3, g is the strong coupling constant related to strong fine structure constant $\alpha_s = g^2/4\pi$. The Parton interaction is larger and the QGP even stronger coupled. Plasmas with such a value of Γ are known to be in liquid phase i.e. the plasma behaves rather like a liquid than a gas due to strong interaction between the charged particles. Thus QGP at least close to the critical temperature is in the liquid phase [3, 4].

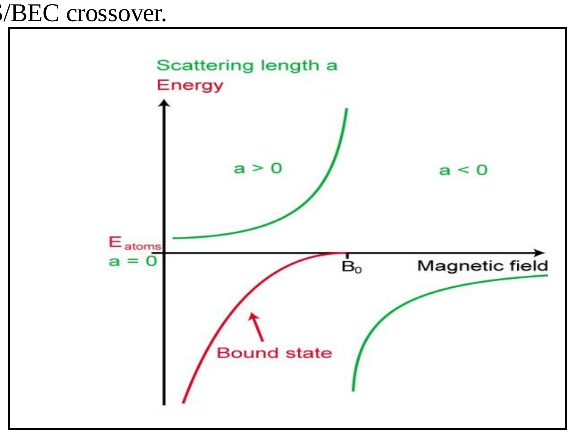
The charge particles within the plasma influences many nearby charged particles within the sphere of influence, called Debye sphere, whose radius is the Debye screening length λ_D. The average number of particles in the Debye sphere is given by plasma parameter $\Lambda = 4\pi n\, \lambda^3_D$. If the strong interaction exists in plasma then the coulomb radius for a particle with an energy $E = q^2/E$ is smaller than the Debye screening length $\lambda_D = 1/m_D$, with m_D is the Debye screening mass. We use non-perturbative estimate $m_D = 6T$ [5] corresponding to Debye screening length less than 0.2 at T = 200 MeV. The estimated distance among quark-gluon plasma constituents for T = 200 MeV is approximately $d = 0.5$ fm $\sim n^{-1/3}$. Also, the coupling constant Γ for strong interaction is estimated in the range (1.5 – 6) related to fine structure constant α_s estimated between (0.2 – 0.5). The estimated values of Coulomb coupling parameter are in the range (1.5 – 6). Taking into account these values range, we can affirm that the quark-gluon plasma has a strongly coupled behavior. In strongly coupled plasma, the interaction range is much larger than the Debye screening length and therefore it cannot be used as a parameter to calculate the transport cross section [6]. This is lie in the case of the interaction of ions with the micro-particles in a complex plasma [7], where ions with an impact parameter larger than the Debye screening length.

2 Feshbach Resonance and the BEC/BCS Crossover

By changing of magnetic field, the interactions between atoms can be controlled over an enormous range. This can be carried out by the coupling of free unbound atoms to a molecular state in which the atoms are tightly bound. The stronger the interaction lies with respect to the energy of two free atoms. The interaction can be described by the scattering length, close to the Feshbach resonance. At resonance, the scattering length diverges and the cross section $\sigma(k) = 4\pi / k^2$ for low momenta k. at Feshbach resonance, the scattering length is,

$$a = a_0\left(1+\frac{\Delta B}{B-B0}\right)$$

Where a_0 is non resonant scattering length, B is the magnetic field, B_0 is the non resonant magnetic field and ΔB is the width. If the scattering length is positive, the molecular energy level is lower in energy than the energy of two unbound atoms. The atoms thus tend to form molecules. If the atoms are fermions, the resulting molecule is a boson. A gas of these molecules can thus undergo Bose-Einstein Condensation (BEC). If the scattering length is negative, isolated molecules are unstable and two fermions form a loosely bound pair, whose size can equal to or even larger than the average distance between particles. A Bose-Einstein Condensation of these fragile pairs is called a BCS state (Cooper pairs). Thus by changing the amount of magnetic field approximately equal to the Feshbach resonance, BEC of tightly bound molecules may cross over to the BCS superfluid of long range pairs [8]. Therefore the unitary limit $a \to\infty$ is also known as the BCS/BEC crossover.



(Figure. 1): BES crossover with an effect under Feshbach Resonance

Some theoretical results have been devoted to understanding a strongly coupled QGP [9,10,11]] in which there exists two temperatures, one for deconfining transition, called chiral-restoration (T_c) and other for vanishing of resonances ($T > T_c$). Such existence of two temperature scales at strong coupling may be naturally understood in the scenario of the crossover from the BCS pairing to the BES (i.e. BCS/BES crossover) [12,13,14]. As Taking into account these values range, we can affirm that the long as the interaction between fermions is weak, the system exhibits the superfluidity characterized by the energy gap in the BCS mechanism. On the other hand, if the attractive interaction is strong enough, the fermions first form bound molecules (bosons), and then they start to condensate into the bosonic zero-mode at some T [8].

The BCS/BES crossover in the QCD contex gives an insight into the strong coupling nature of the quark gluon plasma in either low or high density. The unitary fermions gas is the intermediate of the BCS/BES crossover, where the s-wave scattering length diverges. Such strongly coupled system can be created in the atomic traps with the external magnetic field tuned to the Feshbach resonances. The possible realization of the BCS-BEC crossover in various systems has been theoretically investigated. These include the liquid ^{3}He [15], the trapped alkali atoms [16], and the nuclear matter [17].

3 Formation of transport properties in BCS/BES Crossover

Non-relativistic fermions at unitary behaves as a very good fluid and show some interesting transport properties, including a very small shear viscosity and entropy. The shear viscosity is inversely proportional to the scattering cross-section and becomes minimum at unitarity. Depending on the temperature T and quark chemical potential μ, strongly interacting matter may occur in three distinct phases: the hadronic phase, the QGP and color-superconducting quark matter. The phase transition occurs from the bound state of nuclear matter to the nuclear liquid-gas transition. For temperature ~ 160 MeV and μ ~ 350 MeV, strongly interacting matter is in the hadronic phase. For smaller chemical potential (smaller net-baryon density) and large temperature, the transition becomes crossover and there is no real distinction between hadronic matter and the QGP. At large chemical potential (large baryon density) and smaller temperature, quark matter becomes a color superconductor.

The change of transport and hydrodynamic properties throughout the crossover remains unclarified; this is interesting because it may provide some unified view on the two independent aspects of QGP, i.e., (i) the survival of the bound states above T_c and (ii) the perfect liquidity. It is also worth mentioning that the hydrodynamic properties of the unitary Fermi gas have recently attracted considerable attention both experimentally [18, 19, 20, 21, 22, 23] and theoretically [24, 25]. In our fermion system for strong attractive interactions, the velocity $v = 1/k_f a >> 1$, the fermions are bound into pairs. These pairs form a Boson system with an effective repulsive interaction.

The most interesting feature of the cross over is that the critical temperature in the BEC regime is independent of the coupling of the attraction between fermions. The increase of coupling only affects the boson structure, while the critical temperature is determined by the boson's kinetic energy. At $T = T_c$ of the BEC regime, the chemical potential of the boson $\mu_B = 2\mu$ should be equal to its mass m_B. On the other hand, m_B must be less than 2m for the binding. Thus we have $\mu < m$ in the BEC regime and BCS/BES crossover takes place at $\mu >> T$.

At finite temperature T, strong coupling includes the free energy

$$E\,(T,N_c,\lambda) = [3/4 + 0(1/\lambda)^{3/2}]$$

$E(T,N_c,0)$ where $E(T,N_c,0)$ is the zero coupling ~ $N_c^2 T$, analogous to Stephens-Boltzmann result for black body radiation. The heavy quark potential is associated with Debye radius ~ 1/T [26]. The viscosity of strongly coupled matter was found to be small and the ratio of viscosity to entropy was found to be [27],

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

which is the smallest possible value for an infinite coupling.
In heavy ion collisions, an adiabatic crossing through the resonance converts approximately all atoms into loosely bound Cooper pairs. If the temperature is low, it undergoes Bose-condensation. The question arise here that why we do not observe large fluctuations when the systems crossing QGP-hadronic phase.

4 Thermal fluctuations and perturbation theory

The strength of interactions in classical plasmas is provided by the parameter g^2 which is equal to the ratio between the average potential energy per particle ($\sim e^2n^{1/3}$) and the mean kinetic energy per particle (proportional to T) i.e. $g^2 = e^2n^{1/3}/\mathrm{T} \sim \left[\frac{\kappa}{\mathrm{T}}\right]$, where $\kappa \sim e^2n^{1/3}$ is the scalar considered, T is temperature, n is number density and e is electric charge. For weak interacting plasma $\mathrm{T} >> e^2n^{1/3}$ i.e. $g << 1$. To predict the strong or weak couple plasma, the strength of interaction depends on the magnitude of relevant thermal fluctuations, which depends on their wavelengths. Suppose the amount of interactions at a given scale k of the gauge fields at that scale is $\langle A^2\rangle_k$, the kinetic energy $\sim (\partial A)^2 \sim k^2\langle A^2\rangle$ with the interaction energy $\sim g^2\langle A^2\rangle^2$ [28]. Consider the expansion parameter $\gamma_k = \frac{g^2\mathrm{T}}{\kappa}$, the plasma particles have typical momenta of the order of temperature i.e. $\kappa \sim \mathrm{T}$ and thus fluctuations noticed at $\gamma_\mathrm{T} \sim g^2$. Thus perturbation theory can also works at zero temperature.

5 Color deconfinement in QGP

Another important feature of the QGP is its color deconfinement. In QCD perturbation theory, the quarks and gluons are free particles that can be described by plane waves and all charges are permanently confined to the interior of hadronic scale ~ 1 fm. But asymptotic freedom clears that high transfer momentum are weak, small momentum transfer involves long distance interactions, which are screened by plasma, without being confined to a local region. This property is quite different from the low energy limit of QCD.
The interaction between color charges damped exponentially and thus the induced color field is the result of correlation function of gluon fields. This function can be calculated in perturbation theory at high temperature. When color charges are screened by other charges in the plasma, it has a finite energy and we can say that color charges are formed of that particular energy. The screening mass,

$$m_D^2 = g^2\mathrm{T}^2,$$

is the order of strong coupling, the Debye screening length $1/m_D \sim 1/g\mathrm{T}$ is very short in perturbed QCD. At Fashbach resonance, the magnetic interaction is only weakly screened and thus the magnetic sector of QCD remains unperturbed (even at high temperature).

6 Conclusion

In Bose-Einstein Condensation limit (BEC-limit), fermions formed strongly bound molecules of quantum liquid. At ~ $\mathrm{T_c}$, collective plasma is formed and Coupling constant $\Gamma >>1$. With non-perturbative estimation, Debye screening length is less than0.2 corresponding to T = 200 MeV. Quark gluon plasma behaves strongly coupled for coupling constant parameter Γ is in the range of (1.5 – 6).
By changing magnetic field (~Feshbach resonance) the BCS/BEC crossover can be controlled. The unitary limit for the BCS/BEC crossover is $a \to \infty$ and fermions behave as a fluid. The shear viscosity at unitary become minimum. The BCS/BEC crossover takes place when the quark chemical potential is larger than the quark temperature T (i.e. $\mu >> \mathrm{T}$). Magnetic interaction is weakly screened in plasma and thus magnetic QCD remains unperturbative at Feshbach resonance.

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